

Questions

- 1) Formally prove that $\log(n!) = O(n \log n)$. (20 points)
- 2) Give the asymptotic (“big O”) complexity of the following: (20 points)
 - (a) $T(n)=2T(n/2)+O(n^2)$
 - (b) $T(n)=3T(n/2)+O(n)$

Problem 3: Running Time and Complexity (30 points)

A programming homework is given to a class. Student A designed an algorithm whose running time is $5n^3$ and Student B designed an algorithm whose running time is $25n^2$, where n is the problem size.

- a) [15 points] What is the problem size that Student B’s algorithm perform better than the Student A’s algorithm? Do not write down the answer. Show each step of your solution.

- b) [15 points] Assume that Student A’s algorithm runs on a machine that executes $10n^5$ instructions/second and Student B’s algorithm runs on a machine that executes $10n^4$ instructions/second. What is the smallest problem size that execution time for Student B’s algorithm takes less seconds than Student A’s algorithm. Show each step of your answer. Do not directly write down the answer. Show each step of your solution.

Note: You may assume that execution of an algorithm with problem size n requires execution of n instructions.

Problem 4: Matrix Multiplication (20 points)

Given the following sequence of 4 matrices to be multiplied. Find the most effective parenthesization for it. Show each step of your answer.

Matrices	Dimensions
A_1	50x25
A_2	25x20
A_3	20x10
A_4	10x25